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Heat and Mass Transfer on MHD Boundary Layer Flow of a Chemically Reacting non-Newtonian Fluid over a Stretching Sheet with Suction

G. Sarojamma[1], B. Ramana[*2], K. Vendabai[1]
[1]Department of Applied Mathematics, Sri Padmavati Women's University, Tirupati, Chittoor (Dt), India
[2]Assistant Professor in Maths, Department of BS& H, QIS College of Engg & Tech., Ongole, Prakasam (Dt), India
[*]ramana.bahunadam@gmail.com

Abstract

This paper is focused on the study of effect of heat and mass transfer on chemically reacting boundary layer flow of a Casson fluid over a porous stretching sheet in the presence of a transverse magnetic field. The governing differential equations are transformed by introducing similarity variables and solved numerically by using shooting method. The velocity, temperature and concentration distributions are discussed numerically through graphs and tables for different parameters entering into the problem. It is observed that the Casson parameter decreases the velocity field while the temperature is enhanced with increasing Casson parameter.

Keywords: Mass Transfer, MHD, Casson Parameter, suction.

Introduction

The combined effects of heat and mass transfer with chemical reaction have attracted many researchers due to its wide range of applications in Engineering and Science. Heat and mass transfer occur simultaneously in the processes of drying and evaporation at the surface of a water body, energy transfer in a wet cooling tower and the flow in a desert cooler. The study of magnetohydrodynamic (MHD) flow of an electrically conducting fluid is of considerable interest due to its application in many engineering problems such as MHD generators, plasma studies, nuclear reactors, and geothermal energy extractions. Das et al. (1994) studied the first order chemical reaction effect on the flow past an impulsively started infinite vertical plate with constant heat flux and mass transfer. Anjalidevi and Kandaswamy (1999) considered the heat and mass transfer on steady laminar flow along a semi infinite horizontal plate in the presence of chemical reaction. The Soret and Dufour effects on heat and mass transfer about vertical surfaces in porous media have been studied by Postelnicu (2004). The effect of radiation on boundary layer flow over a moving vertical porous plate was analyzed by Makinde (2005). Muthucumaraswamy et al. (2006) have studied the effect of chemical reaction on moving isothermal vertical plate with radiation. Hossain and Mandal (1985)investigated mass transfer effects on unsteady hydromagnetic free convection flow past an accelerated vertical porous plate. Jha (1991) studied the effect of magnetic field on free convection and mass transfer flow past a uniformly accelerated vertical plate through a porous medium. Elbashbeshy (1997) analyzed the heat and mass transfer along a vertical plate in the presence of magnetic field. The combined heat and mass transfer in MHD free convection flow from a vertical surface with Ohmic heating and viscous dissipation was analyzed by Chen (2004).

The study of boundary layer flow over a stretching sheet has finds applications in chemical engineering, particularly in manufacturing process of artificial film, artificial fibers, polymer extrusion, drawing of plastic films and wires, glass fiber and paper production. Crane (1970) was the first researcher who investigated the boundary layer flow over a stretching surface. After this pioneering work, the study of fluid flow over a stretching sheet has received wide attention among researchers. Rajagopal et al. (1984) discussed the flow of second-order fluid over a stretched sheet. Anderson et al. (1992) considered the effect of magnetic field on the flow of a viscoelastic fluid past a stretching sheet. Abel et al. (2005) analyzed MHD boundary layer flow over continuously moving stretching surface embedded in a porous medium by considering the Buoyancy force and thermal radiation effects. Mukhopadyaya et al. (2008) discussed the free convective boundary layer flow with variable viscosity over a stretching surface with thermal radiation. Pal (2009) investigated the mixed convection flow of an incompressible fluid over a stretching sheet in the presence of radiation. Ahmed (2009) analyzed the free convective heat and mass transfer of a viscous incompressible fluid over a stretching sheet in the presence of suction with Soret and Dufour effects.

Heat and mass transfer in non-Newtonian fluids have applications in engineering such as catalytic reactors, the filtration and blood plasmaphosresis devices. Casson fluid is the most popular non-Newtonian fluid used to model blood. It is defined as a shear thinning liquid which has a infinite viscosity at zero rate of shear, possessing a yield stress below which no flow occurs and zero viscosity at infinite rate of shear. It is reduced to Newtonian fluid at very high wall shear stress i.e. when the wall stress is much greater than yield stress. Merill et al. (1965) and McDonald (1974) conducted experiments on the behaviour of blood as a Casson fluid. Eldabe (1995) considered the heat transfer of Casson fluid flow between two rotating cylinders. The flow of Casson fluid in a tube was studied by Dash et al. (2000) and Nagarani et al. (2004). Mass transfer in a Casson flowing through an annular geometry was examined by Nagarani *et al.* (2006). Attia(2010) analyzed the transient Couette flow of a Casson fluid between parallel plates with magnetic field and heat transfer. The unsteady boundary layer flow of a Casson fluid over a moving flat plate was studied by Mustafa et al. (2011). Hayat et al. (2012) studied the mixed convection stagnation point flow of a Casson fluid. Shehzad (2013) discussed the effects of mass transfer on the MHD boundary layer flow of a Casson fluid with chemical reaction.

In view of the above studies, we consider the MHD boundary layer flow of a Casson fluid over a stretching sheet with heat and mass transfer and chemical reaction.

Mathematical Formulation

Consider the steady, incompressible flow of a Casson fluid over a porous stretching surface at y = 0. Choose the coordinate system such that x-axis is parallel to the surface and y-axis normal to the surface. The fluid occupies half space y > 0. A uniform magnetic field B_0 is applied in the y direction. The transverse applied magnetic field and magnetic Reynolds number are assumed to be very small, so that the induced magnetic field and Hall effects are negligible.We also considered the heat and mass transfer processes in the presence of chemical reaction and suction.

The rheological equation of state for an isotropic and incompressible flow of a Casson fluid can be written as (Nakamura and Sawada (1988), Mustafa et al. (2012))

$$\tau_{ij} = \begin{cases} 2\left(\mu_B + P_y\big/\sqrt{2\pi}\right)e_{ij}, & \pi > \pi_c \\ 2\left(\mu_B + P_y\big/\sqrt{2\pi_c}\right)e_{ij}, & \pi < \pi_c \end{cases}$$

Where μ_B is the plastic dynamic viscosity of the non-Newtonian fluid, P_y is the yield stress of the fluid, e_{ij} denotes the (i, j) - th component of the deformation rate, $\pi = e_{ij}\, e_{ij}$ is the product of the component of deformation rate with itself, π_c is the critical value of π based on the non-Newtonian model.

Under above conditions the continuity, momentum and energy equations are

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}= \nu\left(1+\frac{1}{\beta}\right)\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho}u \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}= \alpha\frac{\partial^2 T}{\partial y^2} \tag{3}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}= D\frac{\partial^2 C}{\partial y^2} - k_1(C-C_\infty) \tag{4}$$

Where u and v are the velocity components in x and y directions, ν is the kinematics viscosity, $\beta = \mu_B\sqrt{2\pi_c}\big/P_y$ is the Casson fluid parameter, σ is the electric conductivity of the fluid, ρ is the density of the fluid, T is the temperature of the fluid, C is the concentration field, α is the thermal diffusivity, D is the mass diffusivity, k_1 is the reaction rate.

The boundary conditions for the velocity, temperature and concentration fields are

$$u = u_w,\ v = -V_0,\ T = T_w,\ C = C_w \quad \text{when } y = 0 \tag{5a}$$

$$u \to 0,\ T \to T_\infty,\ C \to C_\infty \quad \text{as } y \to \infty \tag{5b}$$

where $u_w = c\,x$ is the stretching velocity of the sheet with c (> 0) being the stretching constant. V_0 is the suction velocity,T_w is the temperature of the Sheet.

We define the following similarity variables as

$$\eta = \sqrt{\frac{c}{\nu}}\,y,\ f(\eta) = \frac{\psi}{x\sqrt{c\nu}},\ \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty},\ \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty} \tag{6}$$

where ψ is the stream function with $u = \partial\psi/\partial y$, $v = -\partial\psi/\partial x$, η is the similarity variable

Substituting the non-dimensional variables (6) into the eqns. (2), (3) and (4), we obtain

$$\left(1 + \frac{1}{\beta}\right) f''' + f f'' - (f')^2 - M f' = 0 \tag{7a}$$

$$\theta'' + \Pr f\,\theta' = 0 \tag{7b}$$

$$\phi'' + Sc\,f\,\phi + Sc * K\phi = 0 \tag{7c}$$

The corresponding non-dimensional boundary conditions are

$$f' = 1,\ f = S,\ \theta = 1,\ \phi = 1 \text{ when } \eta = 0 \tag{8a}$$

$$f'(\infty) \to 0,\ \theta(\infty) \to 0,\ \phi(\infty) \to 0 \quad \text{as } \eta \to \infty \tag{8b}$$

where the primes denote the differentiation with respect to η, $M = \frac{\sigma B_0^2}{\rho c}$ is the magnetic parameter, $\Pr = \frac{\nu}{\alpha}$ is the Prandtl number, Sc = $\frac{\nu}{D}$ is the Schmidt number, $K = \frac{k_1}{c}$ is the Chemical reaction parameter, $S = \frac{V_0}{\sqrt{c\nu}}$ is the suction parameter.

The physical quantities of interest are the skin friction coefficient C_f, the local Nusselt number Nu and local Sherwood number Sh which are defined as

$$Re_x^{1/2}\,C_f = \left(1 + \frac{1}{\beta}\right) f''(0)$$

$$Re_x^{-1/2}\,Nu = -\theta'(0)$$

$$Re_x^{-1/2}\,Sh = -\phi'(0)$$

where $Re_x = x\,u_w(x)/\nu$ is the local Reynolds number.

Results and Discussions

The objective of the present analysis is to understand the effect of magnetic field and chemical reaction on the heat and mass transfer in the Casson fluid flow past a vertical stretching surface. The governing equations (7a-c) along with boundary conditions (8a, b) are solved by using Runge-Kutta fourth order shooting technique. The effect of the Casson parameter (β), Magnetic field (M), Suction parameter (S), Prandtl Number (Pr), Schmidt's Number (Sc) on the flow variables have been graphically presented.

Fig 1 presents the effect of Casson parameter (β) on velocity. It is observed that the velocity decreases asymptotically from its highest value on the surfaces to zero as $\eta\to\infty$. The presence of yield stress reduces the velocity. Increasing values of β decreases the velocity further and thus there is a decrease in the thickness of the boundary layer.

Fig.2 shows the variation of magnetic field (M) on velocity**.** The effect of magnetic field on velocity is similar to that of β. Larger the Lorentz force gives the lesser velocity as this acts as a retarding force.

Fig 3 illustrates the effect of Suction parameter (S) on velocity profile. The presence of Suction reduces the thickness of the boundary layer. Increasing values of Suction parameter (S) leads to further reduction in the thickness of the boundary layer. Also the velocity approaches to zero value much faster for higher values of S. When S = 3 there is a double fold reduction in the thickness of the boundary layer to that of the impermeable case.

Fig 4 represents the variation of β on temperature profile. It is observed that the temperature increases with increase in the Casson parameter. However this variation is very nominal.

The variation of magnetic field M on temperature is shown in fig 5. The presence of magnetic field increases the temperature and hence the thickness of the thermal boundary layer, further increase in M results in increase in the thickness of the thermal boundary layer.

The variation of Prandtl number (Pr) is plotted in fig 6. For increase values Pr results in significant reduction in the temperature due to the fact increasing values of Pr amounts to lesser thermal conductivity. When Pr = 3 the thickness of the thermal boundary layer is almost 1/3 of that when Pr = 0.3

Fig 7 gives the effect of suction parameter S on concentration profile. It may be noted that increase in suction parameter leads to reduction in the concentration and thus the thickness of the boundary layer is reduced.

The concentration profile for various values Sc is plotted in Fig 8. It is observed that concentration decreases as increase in Schmidt number.

The influence of Magnetic field M, chemical reaction parameter K and Casson parameters β on concentration profile is given in fig 9-11. The behaviour of the concentration is qualitatively similar with respect to these three parameters increase in M, K and β increases the concentration. However in all the case the variation is very nominal.

The physical quantities viz., the skin friction co-efficient on the wall and the local Nusselt number which are very significant in engineering applications are proportional to the values of $(1+1/\beta)*f''(0)$ and $-\theta'(0)$ respectively and their values are given in the table.

Table 1 presents the values of skin friction on the stretching surface. It is observed that the wall stress reduces with increasing values of yield stress which is inconsistent with the fact that the velocity decrease. The Nusselt number decreases with increasing values of β. The presence of Magnetic field also increases the shear stress. Similarly the effect of magnetic field reduces the Nusselt number. As a suction rate increases it is observed that the skin friction and Nusselt number both increase. The Sherwood number decreases with Casson parameter and the magnetic parameter also reduces the Sherwood number. However the Sherwood number increases with increasing values of suction.

Conclusions

This paper gives the effect of Heat and mass transfer on MHD boundary layer flow for a Casson fluid over a stretching sheet in presence of chemical reaction. The governing equations are solved by shooting technique. It is observed that velocity is decreasing function of Casson parameter β, Magnetic parameter M, Suction parameter S. The thermal boundary layer thickness decreases with an increase in Prandtl number Pr. The effect of β and M on Skin friction coefficient is opposite.

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Table 1: Numerical values of the skin-friction coefficient (1+1/Beta) *f ″ (0), Nusselt number - **θ′(0), Sherwood number - ϕ′(0) for different values of β, M, S**

β	M	S	(1+1/Beta) *f ″ (0)	- θ′(0)	- ϕ′(0)
0.5	0.5	0.5	2.38606	0.187516	0.459373
0.8			2.10406	0.18041	0.403975
1.3			1.89814	0.174778	0.341054
2			1.77069	0.171103	0.283858
∞			1.5	0.162811	0.0384999
0.8	0.0		1.77089	0.187974	0.462378
	0.6		2.16377	0.179231	0.392587
	1.2		2.48886	0.173581	0.32423
	1.5		2.68156	0.170746	0.277312
	0.2	0.0	1.64327	0.158177	0.121227
		0.7	2.03007	0.195931	0.554911
		1.4	2.48607	0.239588	0.953944
		2	2.92354	0.281567	1.29686

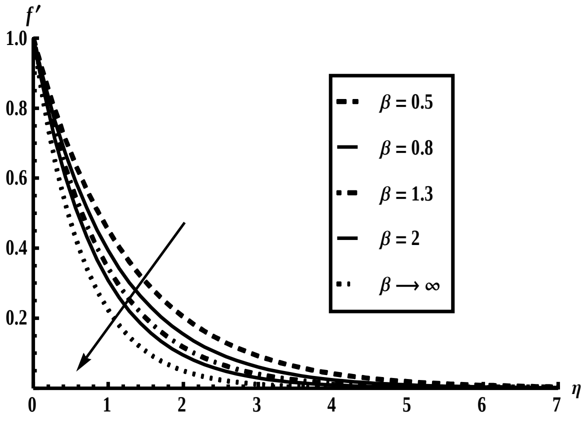

Fig 1. Velocity for different values of β

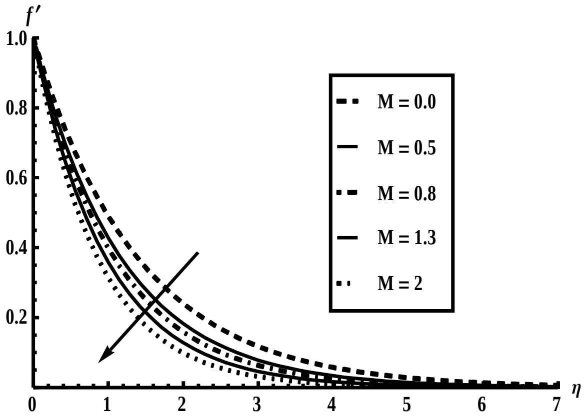

Fig 2. Velocity for different values of Magnetic parameter M

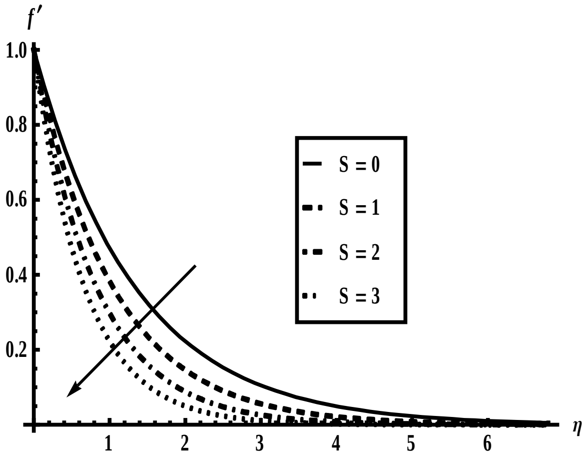

Fig 3. Velocity for different values of suction parameter S

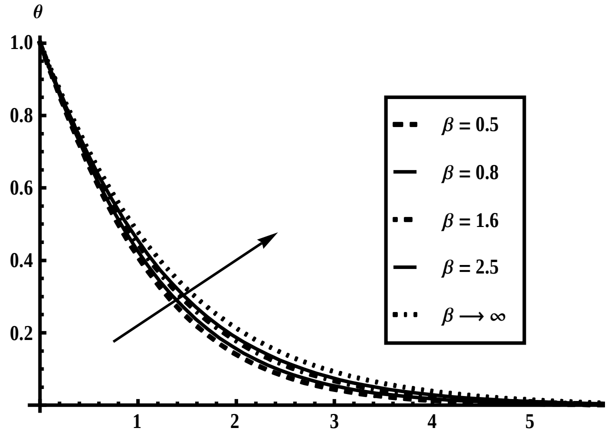

Fig 4. Temparature profiles for different values of β

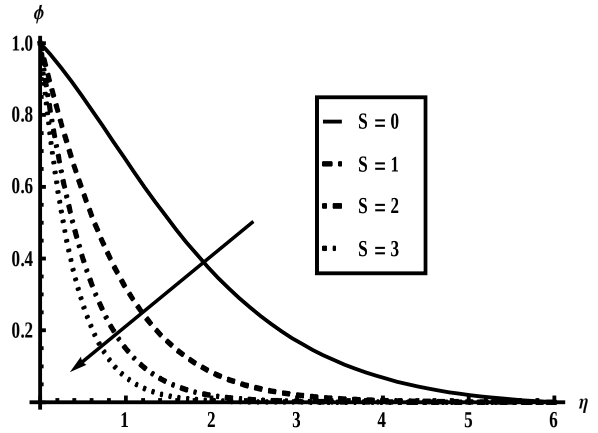

Fig 7. Concentration profiles for different values of suction parameter S

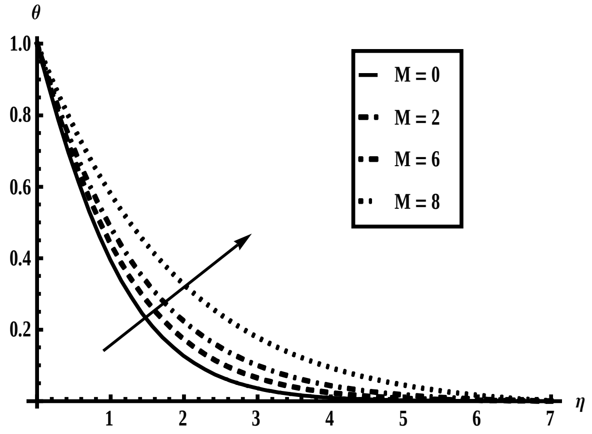

Fig 5. Temparature profiles for different values of M

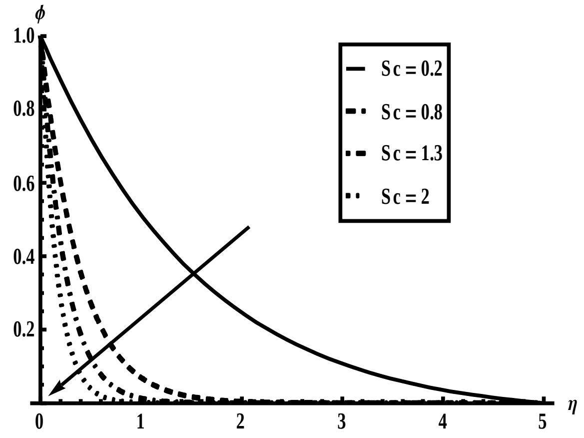

Fig 8. Concentration profiles for different values of Sc

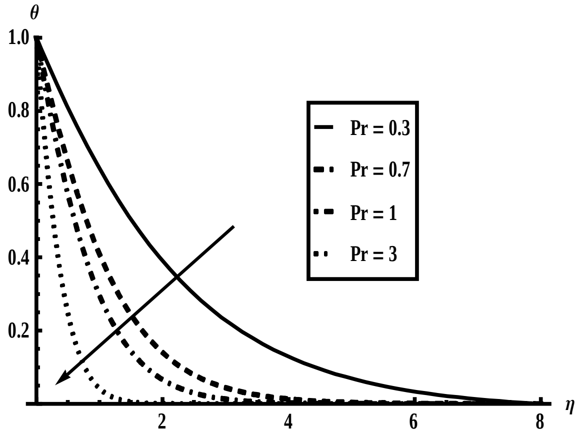

Fig 6. Temparature profiles for different values of Pr

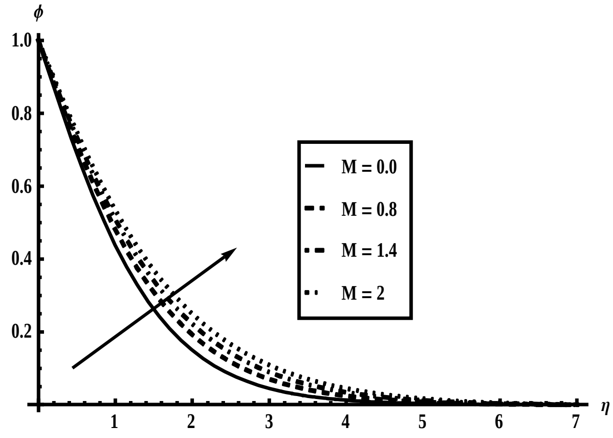

Fig 9. Concentration profiles for different values of M

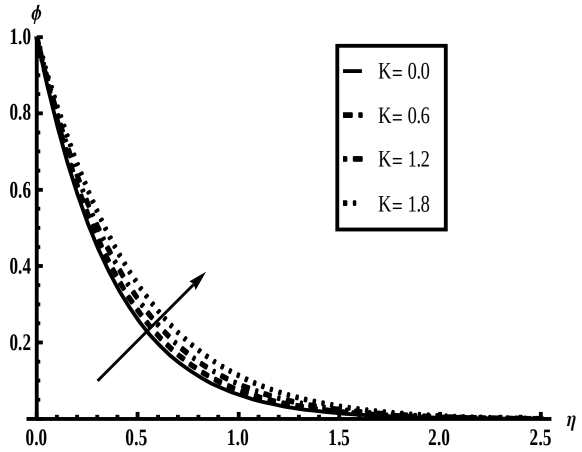

Fig 10. Concentration profiles for different values of chemical reaction parameter K

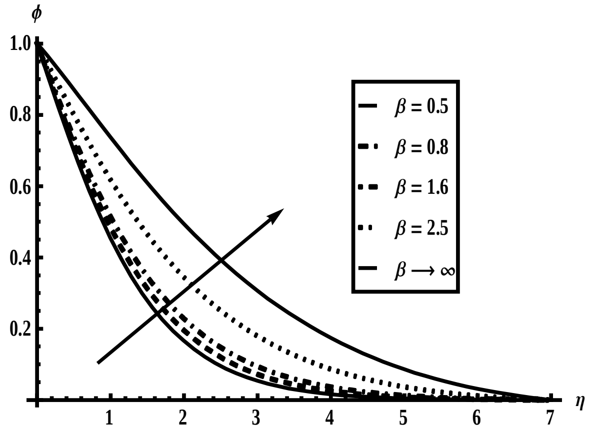

Fig 11. Concentration profiles for different values of β